Planar Transmission Lines—Part III*

The following are a few comments on the above.[1] This paper purports to derive criteria for the optimum line from the standpoint of stability and attenuation. The conclusions arrived at, however, are erroneous.

The stability criterion derived, $0.03 < w/h < 0.75$, where w is the strip width and h is half the ground plane spacing, would limit the characteristic impedance to values greater than 120 ohms which would be most unfortunate if true. The basis for this choice is the rapid variation of $f(k^2)$ in the neighborhood of $k=0$ or 1. In the neighborhood of $k=0$, however,

$$f(k^2) \approx 4\left(\ln 2 + \frac{\pi w}{4h}\right)$$

so that it does not vary rapidly with the line dimensions and the upper limit on w/h is not necesasary.

An examination of stability might better be made by directly investigating the dependence of characteristic impedance on the dimensions. In the case of low impedance lines, $w \gg h$ and

$$Z_0 = \frac{15\pi^2}{\sqrt{k}} \frac{1}{\ln 2 + \frac{\pi w}{4h}} \approx \frac{60\pi}{\sqrt{k}} \frac{h}{w}.$$

Thus

$$\frac{dZ_0}{Z_0} = \frac{dh}{h} - \frac{dw}{w}$$

which indicates that low impedance lines are not unstable. For high impedance lines, $w \ll h$ and

$$Z_0 = \frac{60}{\sqrt{k}} \ln \frac{16h}{\pi w}.$$

In this case,

$$\frac{dZ_0}{Z_0} = \frac{60}{Z_0}\left(\frac{dh}{h} - \frac{dw}{w}\right)$$

which indicates that very high impedance lines are even more stable. This latter condition is generally true for any transmission line operating in the TEM mode as is obvious from the logarithmic dependence of the characteristic impedance on the line dimensions.

The statement that the attenuation decreases as w is decreased is contrary to fact, both experimental[2] and theoretical.[3] This error arose by neglecting the dependence of k^2 on the ratio w/h in (11) and (12). The dependence of the attenuation on w for a single strip line may be found from (33) of Park's original paper.[4] This is[4]

$$\alpha \approx \frac{\eta\sqrt{k}}{120\pi h} \frac{4h}{\pi w}\left(\ln \frac{4h}{\pi t} + \frac{\pi w}{2h}\right); \quad \frac{\pi w}{4h} \gg \ln 2$$

$$\approx \frac{\eta\sqrt{k}}{120\pi h}\left(\frac{4h}{\pi w} \ln \frac{4h}{\pi t} + 2\right).$$

Then

$$\frac{\partial \alpha}{\partial w} = -\frac{\eta\sqrt{k}}{30\pi^2 w^2} \ln \frac{4h}{\pi t},$$

showing that α decreases as w increases. This is also true for the case of a balanced strip line.

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[1] D. Park, "Planar transmission lines—III," IRE TRANS., vol. MTT-4, p. 130; April, 1956.

[2] E. G. Fubini, W. E. Fromm and H. Keen, "New techniques for high-Q strip microwave components," 1954, IRE CONVENTION RECORD, Part 8, p. 91.

[3] S. Cohn, "Problems in strip transmission lines," IRE TRANS., vol. MTT-3, p. 119, March, 1955.

[4] D. Park, "Planar transmission lines," IRE TRANS., vol. MTT-3, p. 1[illegible], April, 1955.

Author's Comment, Part IV[5]

In this note the characteristic impedance will be estimated for a transmission line composed of two wires of radius a and separation $2d$, embedded in a dielectric between two broad conducting strips which are a distance $2h$ apart and which extend for some distance beyond the wires on each side. For the notation and ideas, as well as a discussion of what is meant by "some distance," we refer to a previous paper.[4] The calculation is done by the method of images, and the approximation will be made that the radius of the wire is very small compared to the other dimensions of the system. If this is true, the charge distribution on each wire and its images will be nearly symmetrical about the center, and the work necessary to bring a unit charge from infinity to the surface of one of the wires is $(\lambda/2\pi\epsilon) \ln (2d/a)$ when only the fields of the two wires are taken into account. Considering next the fields due to the nth image of the pair of wires, located at a distance $2nh$ above them, we see easily that the corresponding work is $(-)^n \frac{1}{2}(\lambda/2\pi\epsilon) \ln [1+(d/nh)^2]$. Summing over all these pairs of images and including the original wires we get

$$V = \frac{\lambda}{2\pi\epsilon} \ln \left\{ \frac{[1 + (d/2h)^2][1 + (d/4h)^2][1 + (d/6h)^2] \cdots}{[1 + (d/h)^2][1 + (d/3h)^2][1 + (d/5h)^2] \cdots} \cdot \frac{2d}{a} \right\}.$$

The infinite products converge and can be evaluated by means of the well-known products for sines and cosines to give

$$V = \frac{\lambda}{2\pi\epsilon} \ln \left\{ \frac{2h}{\pi d} \frac{\sinh (\pi d/2h)}{\cosh (\pi d/2h)} \cdot \frac{2d}{a} \right\}.$$

The potential of a point on the other wire is the negative of this, and so the total potential difference is twice V. The characteristic impedance in the TEM mode is given by $Z = \sqrt{(\mu\epsilon)}/C$ where C, the line's capacity per unit length, is λ/V. Thus $Z = 2\sqrt{(\mu\epsilon)}\, V/\lambda$, and

$$Z = \frac{120}{\sqrt{\kappa}} \ln \left\{ \frac{4h}{\pi a} \tanh \frac{\pi d}{2h} \right\}.$$

This is the same as the equation before (22a),[4] if w there is replaced by $4a$. Thus it results that a flat central strip in a line of this kind is equivalent to (but more lossy than) a wire whose diameter is half the width of the strip.

Finally, a previous note in this journal[1] which purported to derive several restrictions on the dimensions of practical transmission lines must be amended. In fact the conditions imposed there were far too stringent, due to neglect of the fact that though the operating characteristics depend somewhat steeply on the parameter k^2 when k^2 is near the ends of its range, this does not mean that it depends very critically on the dimensions of the line, as can in fact easily be seen from the relevant formulas.[4] The restrictions are thus unnecessary. I would like to thank K. S. Packard for calling my attention to this mistake. Further, the numerical coefficients in (11) and (12)[1] should be corrected to read $7.4 \times 10^{-4}/b$ and $3.7 \times 10^{-4}/b$ respectively.

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[5] Received by the PGMTT, October 10, 1956.

A Low VSWR Matching Technique*

I would like to call your attention to a certain error in the above correspondence.[1] The admittance function plotted by the author on the Smith chart winds counterclockwise. This is contrary to the laws of nature for passive linear networks. These laws prescribed that the plot of the admittance function wind clockwise from the low to the high frequency.

Rotation of the admittance function to the generator would result in a loop instead of a point when the high-frequency end catches up with the low-frequency end. Furthermore, continued rotation causes f_h to pass f_l which is contrary to the author's figures.

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[1] P. A. Rizzi, IRE TRANS., vol. MTT-4, pp. 185–186; July, 1956.

Author's Comment[2]

In view of Mr. Goldstone's comment, the curvature of curves B and B' in Figs. 1 and 2 should be changed from concave with respect to the center of the chart to convex. Rotation of the resultant curve to the $g=1$ circle will, as Mr. Goldstone points out, yield an admittance plot which contains a small loop.

Nevertheless, the general matching procedure as outlined is still valid.

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[2] Received by the PGMTT, November, 1956.